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Transmission Line on Semiconductor Substrate with Distributed Amplification

Michal POKORNÝ, Zbyněk RAIDA

Department of Radio Electronics, Brno University of Technology, Purkyňova 118, 612 00 Brno, Czech Republic

pokornym@feec.vutbr.cz, raida@feec.vutbr.cz

Abstract. In order to compensate losses in metal strips, an active microstrip line on a semiconductor substrate is proposed, and its finite element model is presented. The active medium is provided by A^3B^5 semiconductor in highintensity electric field. The simple model of the active media is developed and used for calculation of the propagation properties of the fundamental mode and for the thermal analysis of the device. The problem of system self-oscillation is discussed and empirical stability criteria are introduced. A proper heat-sink is proposed to provide the operation in a continuous regime.

Keywords

GaAs, active, mictrostrip, millimeter-wave, Gunn's effect, thermal, COMSOL.

1. Introduction

The output power of the convectional discrete semiconductor devices drastically drops at millimeter-wave frequencies due to their fundamental limitations. The first one is associated with the device capacitance which results in impedance decreasing with frequency and creates a problem to match the device with a waveguides. Next, the maximal frequency at which a device acts as an active element corresponds to the transit time of charge carriers or high field domains through the structure. Both mentioned problems can be suppressed by the reduction of devices in size. Besides, the minimal dimensions are also limited by several factors; the reduced device surface makes it difficult to effectively dissipate heat [1].

To overcome these fundamental limitations on output power, the performance of active components should be based on the principles of distributed interaction of the electromagnetic wave with active medium [1], [2]. As the promising active medium, the bulk negative differential resistance (BNDR) can be considered. BNDR arises in $A^{3}B^{5}$ semiconductors (e.g. GaAs) due to inter-valley scattering of hot electrons in high electric fields. While the wavelength is short enough for the effective interaction with the bulk semiconductor, the traveling wave amplification is possible [2].

Some authors suggest the concept of active transmission lines combining the technology of monolithic microwave integrated circuits with the traveling wave amplification provided by a semiconductor substrate. Up to date, mainly experimental results were published in this field. The first successful measurements of electromagnetic wave amplification by BNDR were performed at the frequency 15 GHz in coplanar microstrip waveguides on the GaAs substrate [3]. Other experiments deal with the fin-line and strip line structures at the frequencies about 40 GHz and 160 V bias voltage in the pulse regime [2]. All published structures faced with the problem of device overheating in the continuous regime, so the problem of the heat-sink is very important. In theoretical works on active structures based on BNDR [2], [4], a complicated equation system describing wave propagation and its possible analytical solution were suggested, but no calculations were performed. The paper [5] deals with the finite element modeling of active dielectric waveguides based on BNDR, however the issue of thermal heating and system stability are not included.

In this paper, a numerical model of the active BNDR based microstrip line is developed and its performance and limits in a continuous regime are investigated. The original contribution is represented by the introducing of the simplifying assumptions which allow us to derive a simple model of the active GaAs medium consisting of the analytical form of the differential conductivity tensor. Then we set the procedure of the BNDR active device analysis which benefits from the separate calculation of active medium parameters and wave propagation. Such procedure is easy to implement in existing commercial software thus enables design of BDNR devices in convectional industry engineering work-flow. Finally the device performance (transmission, thermal) is investigated by calculations using COMSOL Multiphysic computational environment which is based on finite element method. The proposed active microstrip line is designed with respect to empirical criteria of system stability in order to avoid self-oscillation.

2. Model of Active Medium

There are many mechanisms affecting the wave propagation in bulk semiconductors. However at millime-

ter-wave frequencies, only free charge concentration is relatively significant, so the diffusive processes and the space-charge wave are involved. The static and transient behavior of semiconductor devices can be described by a conventional drift-diffusion scheme [6]. The scheme should be extended by the Helmholtz equation to define the wave propagation. The resultant system basically describes two kinds of waves, the slow space-charge wave with speed given by carriers drift velocity, and fast wave with speed of light [2]. Since the space charge is present at interfaces and defects of uniformly doped semiconductor only, we neglect the slow space-charge wave and assume the fast wave only. Then, we can introduce the simplification by the separate calculation of media properties and wave propagation in order to obtain an easily computable model of electromagnetic wave propagation in BNDR media. The illustrative description of such model follows.

The discussed structure is depicted in Fig. 1. The sample of n-doped GaAs is biased via ohmic contacts by an external voltage source which excites the electrostatic field \mathbf{E}_0 in volume of a sample. The harmonic propagation of linearly polarized wave \mathbf{E}_p is superposed on \mathbf{E}_0 , so the total electric field intensity is given by their addition.



Fig. 1. Illustrative sketch of wave propagation in biased GaAs sample.

If the doping profile of the GaAs sample is uniform and low perturbation of free space charge by \mathbf{E}_p is assumed, the gradient of the carrier concentration is zero; thus the diffusive processes can be neglected. Then the free carrier concentration c_n can be assumed to be always equal to the doping concentration N_D (all impurities are ionized). Under these simplifying considerations, the properties of the medium can be simply described by the conductivity σ given by

$$\sigma = q N_D \mu_n. \tag{1}$$

Here, q denotes the elementary charge and μ_n is the electron mobility.

If the external voltage is applied and the electrostatic field \mathbf{E}_0 arises, the conductivity becomes to be anisotropic due to the reduction of the mobility caused by the transfer

of electrons from the lowest valley in the conduction band to the valleys of higher energy (Gunn's effect). For bulk GaAs, Gunn's effect can be described by [6]

$$\mu_{n}^{E} = \frac{\mu_{n} + v_{sat} \frac{(E_{0})^{3}}{(E_{crit})^{4}}}{1 + \left(\frac{E_{0}}{E_{crit}}\right)^{4}}$$
(2)

Here, μ_n represents the original mobility, v_{sat} is the saturation velocity, E_0 is the component of \mathbf{E}_0 in the direction of current flow and E_{crit} is the threshold electric field. As a result, the conductivity is expressed as a uniaxial tensor $\underline{\sigma}$. In order to obtain small-signal approximation, $\mathbf{E}_0 \ll \mathbf{E}_p$ is assumed [2]. That introduces the differential form of the conductivity $\underline{\sigma}_D$.

Instead of using the piecewise linear approximation of the differential conductivity supposed in [2] which is invariant with electric field and doping concentration, we derived the analytical form which can be computed from the drift component of current density formula [6]

$$J_0 = q N_D \mu_n^E E_0 av{3}$$

Substituting the mobility formula (2) into (3) and differentiating with respect to the electric field intensity, the analytical form of the differential conductivity is obtained

$$\sigma_D(E_0) = \frac{qN_D E_{crit}^{4} \left(4v_{sat} E_0^{3} - 3\mu_n E_0^{4} + \mu_n E_{crit}^{4}\right)}{\left(E_0^{4} + E_{crit}^{4}\right)^2} \quad (4)$$

The formula is advantageous because it can be evaluated for various temperatures, doping concentrations etc. by proper modeling of the physical quantities of GaAs.

Finally, the elements of the desired tensor $\underline{\sigma}_D$ are given by (4) evaluated for the corresponding component of the electrostatic field \mathbf{E}_0

$$\underline{\sigma}_{D} = \begin{bmatrix} \sigma_{D}(E_{x0}) & & \\ & \sigma_{D}(E_{y0}) & \\ & & \sigma_{D}(E_{z0}) \end{bmatrix}$$
(5)

If the Poisson equation is solved in the domain of the active semiconductor medium, then the solution defines the components of the biasing electric field and the differential conductivity tensor can be evaluated and used for the purpose of wave propagation simulations in BNDR devices. Such medium description is used in the following calculation of the active transmission line.

2.1 Thermal Properties of GaAs

The main physical parameters of semiconductors significantly depend on the temperature. The interaction of electrons with the thermally generated vibrations of the crystal lattice can be simply modeled by the reduction of carrier mobility [6]

$$\mu_n^L = \mu_n^{300} \left(\frac{T}{300 \text{K}}\right)^{-\alpha} \,. \tag{6}$$

The dependency of the saturation of carrier drift velocity on the temperature is given by [6]

$$v_{sat} = \frac{v_{sat}^{300}}{\left(1 - A_n\right) + A_n\left(\frac{T}{300K}\right)}$$
 (7)

A sufficient approximation of the thermal conductivity dependence can be modeled by [6]

$$k_T = k_{T300} \left(\frac{T}{300 \text{K}}\right)^{-\beta} .$$
 (8)

The generated heat for a non-degenerated semiconductor is given by [6]

$$H = J_0 E_0. \tag{9}$$

In equations (6) to (9), the quantities with index $_{300}$ denote the values at 300 K; coefficients α , β and A_n are obtained by fitting experimental data and *T* is the temperature. Typical values of physical quantities of GaAs are given in Tab. 1 [6].

μ_n^{300}	$0.8 \ [m^2 V^{-1} s^{-1}]$	k_{T300}	46 [Wm ⁻¹ K ⁻¹]
α	1 [-]	β	1.25 [-]
v_{sat}^{300}	72·10 ³ [ms ⁻¹]	A_n	0.56 [-]

Tab. 1. Physical quantities of GaAs.

The evaluated differential conductivity (4) for the donor concentration $N_D = 10^{19} \text{ m}^{-3}$ and various temperatures is depicted in Fig. 2. Obviously, the BNDR behavior of the medium arises at 340 kVm⁻¹, and the strongest BNDR is reached at about 480 kVm⁻¹ of the electric field magnitude.



Fig. 2. Differential conductivity characteristics of the bulk GaAs at different temperatures.

3. Active Microstrip Line Calculations

The dimensions of the investigated structure were adopted from [7]. The structure represents the 50 Ω microstrip line on a GaAs wafer with the thickness of 0.1 mm and relative permittivity 13. The strip is made of gold with conductivity of 45.2 MSm⁻¹, and transverse dimensions are 70 × 4 μ m². The 2D finite-element model of the microstrip line was implemented in COMSOL Multiphysics. Since the finite-element models have to be enclosed, the microstrip line was inserted into the waveguide 2.54 × 2.54 mm² with walls defined as the perfect electric conductor. The size of such a waveguide is more than ten times larger than the thickness of the substrate in order to minimize its influence on the microstrip line parameters [8].

If sufficient voltage for creating a BNDR is applied, it may lead to current instabilities (self oscillations). The stable operation of the device can be achieved if the electron concentration c_n (N_D respectively) and the distance between the ohmic contacts (thickness of substrate *h*) are chosen according to the criterion [2]

$$N_D h < 10^{15} . (10)$$

This criterion implies that the maximum value of the donor concentration N_D for a given h is 10^{19} m^{-3} .

First, the validity of the model was verified by calculations of the attenuation by ohmic losses for the dominant mode using an analytical formula [8] based on the conformal mapping method (CMM) and by direct comparison with experimental data of the same microstrip line on a GaAs substrate presented in [7]. For this purpose, the model of a passive microstrip line on a lossless substrate was analyzed ($N_D = 0 \text{ m}^{-3}$). Fig. 3 shows good agreement of the numerical, analytical and experimental results.



a) HTC = $100 \text{ kWm}^{-2}\text{K}^{-1}$, b) HTC = $10 \text{ kWm}^{-2}\text{K}^{-1}$.

Then, the model of the active version was analyzed with the uniformly doped substrate. However, the presence

of impurity increases its conductivity, so the effect of BNDR is suppressed by additional losses in the semiconductor. The calculation of resistive heating by propagating wave in the cross section of the device (see Fig. 4) reveals the high-loss areas located straight under the metal line edges. If these areas are kept undoped or doped lightly, the loss in the substrate can be significantly reduced. Since the transition between two doping concentrations cannot be arbitrarily steep due to the limitations of the manufacturing technology and natural diffusion of impurities in bulk material, we use a typical soft transition with the doping concentration difference of two orders. Fig. 5 shows the used doping profile, where the substrate doping concentration is 10^{17} m⁻³ and the active area is formed by a highly doped region with the donor concentration 10^{19} m⁻³. The active area is located straight under the metal strip and is narrow enough not to overlap the high-loss areas.

The simulation results of the described structure for two heat dissipation conditions (described later) at bias voltage 50 V are depicted in Fig. 3 in comparison with the results of passive model. The significant reduction of attenuation is observed, even the gain is provided at lower frequencies, where the total losses are lower than the amplification provided by the semiconductor substrate.



Fig. 4. Distribution of resistive heating [Wm⁻¹] inside the GaAs substrate uniformly doped by donors at concentration 10¹⁹ m⁻³.





The stepwise description of presented calculations in COMSOL Multiphysics follows.

First, the Poisson equation is solved while the bias voltage is applied on the gold strip. The ground plane was set to zero potential. Solution of the static electric field \mathbf{E}_0 is depicted in Fig. 6. Obviously, the *z* component is dominant under the gold strip in the substrate.

Then, the heat generation is evaluated and the heat transport problem is solved in the domain of the GaAs substrate. The heat transfer coefficient (HTC) is set to $5 \text{ Wm}^{-2}\text{K}^{-1}$ at boundaries where the air without convection is assumed [9]. The efficient heat sink is provided at the ground plane interface, so the HTC is set to 100 kWm⁻²K⁻¹ at this boundary. The ambient temperature is set to 300 K. The heat generation is given by equations (3) and (9). The solution of the temperature distribution in the continuous operation regime is depicted in Fig. 7. The peak temperature

ture is 400 K while the maximum working temperature of GaAs is about 723 K [8], so the device can be functional when the used heat sink provides a sufficient heat transfer.



Fig. 6. The solution of Poisson equation. Surface: Electric field intensity, z component [kVm⁻¹] Arrow: Electric field intensity, transversal vector.



Fig. 7. Temperature distribution in continuous regime. Surface: temperature [K]. Arrow: Heat flux vector.

Next, the differential conductivity tensor $\underline{\sigma}_D$ is evaluated using the current solution \mathbf{E}_0 and formula (5) with respect to the computed temperature distribution. Fig. 8 represents the resultant distribution of *z* component of $\underline{\sigma}_D$. The active region exhibits by BNDR up to -0.38 Sm^{-1} . The maximal conductivity does not exceed the value 0.0128 Sm^{-1} , so the additional loss in the substrate is lower than the gain in the active region.



Fig. 8. Distribution of differential conductivity, *z* component [Sm⁻¹].

Finally, the eigenvalue solver is used to find the modes propagating at a certain frequency. Only the fundamental hybrid mode exists up to frequency 56 GHz. From this frequency, the second mode given by shielding PEC waveguide dimensions starts to propagate. The electric field distribution of the fundamental mode at 25 GHz is presented in Fig. 9.

The temperature and attenuation variation with the applied bias voltage at 25 GHz is depicted in Fig. 10. The ohmic losses in doped GaAs cause the extreme attenuation

of the transmission line until the sufficient voltage is applied to create the BNDR domain in the substrate. Then, the attenuation is significantly reduced with the increasing voltage to saturate minima at 50 V. The temperature is increased exponentially at a lower bias voltage and changes gradually to linear dependency.



Fig. 9. Electric field intensity of the fundamental mode at 25 GHz. Surface: Electric field intensity, *x* component [Vm⁻¹] Arrow: Electric field intensity, transversal vector.



Fig. 10. The dependency of temperature and attenuation on applied bias voltage.

Other computations were done for HTC coefficient equal to $10 \text{ kWm}^{-2}\text{K}^{-1}$ in order to investigate the effect of a less efficient heat sink. Even in this case, the maximum device temperature 557 K does not exceed the limit for the GaAs material. The limit is reached at 70 V of the bias voltage. The comparison of device performance at different cooling conditions can be seen in Fig. 3 and Fig. 10.

4. Conclusions

In the paper, the active microstrip line was proposed in order to compensate the ohmic losses which are the serious problem of transmission lines in millimeter-wave frequency band. The model of the active medium represented by the tensor of the differential conductivity was deduced for A^3B^5 semiconductor materials and analytically formulated for GaAs using the empirical formula of the electron mobility reduction in high intensity electric field.

The finite element model of an active microstrip line was analyzed in COMSOL Multiphysics. The attenuation of the fundamental mode and thermal characteristics of an active device were calculated and discussed. The results show the significant reduction of attenuation with the applied bias voltage. Even the gain was observed at lower frequencies about 5 GHz. The thermal problem requires the efficient heat sink defined by the transport coefficient. The optimal value that guarantees the proper heat dissipation is about $10 \div 100 \text{ kWm}^{-2}\text{K}^{-1}$.

Further work will be focused on the experimental verification of presented calculations and development of more complex devices as active antennas and active feeding networks. Recently, another physical mechanism, which can provide the traveling wave amplification at lower electric fields, seems to be more perspective e.g. the slow-wave interaction with drifting charge flow [10].

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About Authors ...

Michal POKORNÝ was born in 1983. He is PhD student at the Dept. of Radio Electronics, Brno University of Technology. His research interests are antenna design and modeling of microwave semiconductor devices.

Zbyněk RAIDA received Ing. (M.Sc.) and Dr. (Ph.D.) degrees from the Brno University of Technology (BUT) in

1991 and 1994, respectively. Since 1993, he has been with the Dept. of Radio Electronics of BUT as the assistant professor (1993 to 98), associate professor (1999 to 2003), and professor (since 2004). From 1996 to 1997, he spent 6 months at the Laboratoire de Hyperfrequences, Universite Catholique de Louvain, Belgium as an independent researcher. Prof. Raida has authored or coauthored more than 80 papers in scientific journals and conference proceedings. His research has been focused on numerical modeling and optimization of electromagnetic structures, application of neural networks to modeling and design of microwave structures, and on adaptive antennas. He is a member of the IEEE Microwave Theory and Techniques Society. From 2001 to 2003, he chaired the MTT/AP/ED joint section of the Czech-Slovak chapter of IEEE. In 2003, he became the Senior Member of IEEE. Since 2001, Prof. Raida is editorin-chief of the Radioengineering journal (publication of Czech and Slovak Technical Universities and URSI committees).